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A VARIABLE EXPONENT BOUNDEDNESS OF THE STEKLOV OPERATOR

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Abstract. In this paper, a sufficiency condition for boundedness of the Steklov operator

$$S_h f(x) = \frac{1}{h}\int_x^{x+h} f(t)dt, \quad h>0$$

has been proved in variable exponent Lebesgue space $L^{p(.)}(0,\infty)$. Here an infinite interval $(0,\infty)$ has been considered with a new decay condition on infinity. A finite interval $[0,2\pi]$ case with a local log- regularity condition has been studied previously in order to be applied on approximation problem.

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