

CONTINUITY OF GENERALIZED RIESZ POTENTIALS FOR DOUBLE PHASE FUNCTIONALS

TAKAO OHNO AND TETSU SHIMOMURA

Abstract. In this note, we are concerned with the continuity of generalized Riesz potentials $I_{p,\mu,\tau,f}$ of functions in Morrey spaces $L^{\Phi,\nu,\kappa}(X)$ of double phase functionals over bounded non-doubling metric measure spaces.

Mathematics subject classification (2020): 31B15, 46E35.

Keywords and phrases: Riesz potentials, Morrey spaces, double phase functionals, continuity, non-doubling measure.

REFERENCES

- [1] P. BARONI, M. COLOMBO AND G. MINGIONE, *Regularity for general functionals with double phase*, Calc. Var. Partial Differential Equations **57** (2018), no. 2, paper no. 62, 48 pp.
- [2] A. BJÖRN AND J. BJÖRN, *Nonlinear potential theory on metric spaces*, EMS Tracts in Mathematics, 17. European Mathematical Society (EMS), Zurich, 2011.
- [3] S. S. BYUN, S. LIANG AND S. ZHENG, *Nonlinear gradient estimates for double phase elliptic problems with irregular double obstacles*, Proc. Amer. Math. Soc. **147** (2019), 3839–3854.
- [4] M. COLOMBO AND G. MINGIONE, *Regularity for double phase variational problems*, Arch. Rat. Mech. Anal. **215** (2015), 443–496.
- [5] M. COLOMBO AND G. MINGIONE, *Bounded minimizers of double phase variational integrals*, Arch. Rat. Mech. Anal. **218** (2015), 219–273.
- [6] ERIDANI, H. GUNAWAN, E. NAKAI AND Y. SAWANO, *Characterizations for the generalized fractional integral operators on Morrey spaces*, Math. Ineq. Appl. **17** (2014), no. 2, 761–777.
- [7] C. DE FILIPPIS AND G. MINGIONE, *Manifold constrained non-uniformly elliptic problems*, J. Geom. Anal. **30** (2020), no. 2, 1661–1723.
- [8] C. DE FILIPPIS AND G. PALATUCCI, *Hölder regularity for nonlocal double phase equations*, J. Differential Equations **267** (2019), no. 1, 547–586.
- [9] P. HARJULEHTO AND P. HÄSTÖ, *Boundary regularity under generalized growth conditions*, Z. Anal. Anwend. **38** (2019), no. 1, 73–96.
- [10] P. HÄSTÖ AND J. OK, *Calderón-Zygmund estimates in generalized Orlicz spaces*, J. Differential Equations **267** (2019), no. 5, 2792–2823.
- [11] F.-Y. MAEDA, Y. MIZUTA, T. OHNO AND T. SHIMOMURA, *Boundedness of maximal operators and Sobolev's inequality on Musielak-Orlicz-Morrey spaces*, Bull. Sci. Math. **137** (2013), 76–96.
- [12] F.-Y. MAEDA, Y. MIZUTA, T. OHNO AND T. SHIMOMURA, *Sobolev's inequality inequality for double phase functionals with variable exponents*, Forum Math. **31** (2019), 517–527.
- [13] Y. MIZUTA, E. NAKAI, T. OHNO AND T. SHIMOMURA, *Campanato-Morrey spaces for the double phase functionals*, Rev. Mat. Complut. **33** (2020), 817–834.
- [14] Y. MIZUTA, E. NAKAI, T. OHNO AND T. SHIMOMURA, *Riesz potentials and Sobolev embeddings on Morrey spaces of variable exponent*, Complex Var. Elliptic Equ. **56**, no. 7–9, (2011), 671–695.
- [15] Y. MIZUTA, T. OHNO AND T. SHIMOMURA, *Sobolev's theorem for double phase functionals*, Math. Ineq. Appl. **23** (2020), 17–33.
- [16] C. B. MORREY, *On the solutions of quasi-linear elliptic partial differential equations*, Trans. Amer. Math. Soc. **43** (1938), 126–166.

- [17] J. MUSIELAK, *Orlicz spaces and modular spaces*, Lecture Notes in Math. **1034**, Springer-Verlag, 1983.
- [18] S. NAGAYASU AND H. WADADE, *Characterization of the critical Sobolev space on the optimal singularity at the origin*, J. Funct. Anal. **258** (2010), no. 11, 3725–3757.
- [19] T. OHNO AND T. SHIMOMURA, *Sobolev inequalities for Riesz potentials of functions in $L^{p(\cdot)}$ over nondoubling measure spaces*, Bull. Aust. Math. Soc. **93** (2016), 128–136.
- [20] C. PERÉZ, *Sharp L^p -weighted Sobolev inequalities*, Ann. Inst. Fourier (Grenoble) **45** (1995), 809–824.
- [21] E. PUSTYLNİK, *Generalized potential type operators on rearrangement invariant spaces*, Israel Math. Conf. Proc. **13** (1999), no. 3, 161–171.
- [22] Y. SAWANO AND T. SHIMOMURA, *Sobolev embeddings for Riesz potentials of functions in non-doubling Morrey spaces of variable exponents*, Collect. Math. **64** (2013), 313–350.
- [23] Y. SAWANO AND T. SHIMOMURA, *Boundedness of the generalized fractional integral operators on generalized Morrey spaces over metric measure spaces*, Zeit. Anal. Anwend. **36** (2017), 159–190.
- [24] Y. SAWANO AND T. SHIMOMURA, *Generalized fractional integral operators over non-doubling metric measure spaces*, Integral Transforms and Special Functions, **28** (2017), 534–546.
- [25] Y. SAWANO, S. SUGANO AND H. TANAKA, *Orlicz-Morrey spaces and fractional operators*, Potential Anal. **36** (2012), no. 4, 517–556.
- [26] P. SHIN, *Calderón-Zygmund estimates for general elliptic operators with double phase*, Nonlinear Anal. **194** (2020), 111409, 16 pp.
- [27] V. V. ZHIKOV, *Averaging of functionals of the calculus of variations and elasticity theory*, Izv. Akad. Nauk SSSR Ser. Mat. **50** (1986), 675–710.