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APPLICATIONS OF SECTIONS AND HALF VOLUMES IN STABILITY

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Abstract. It is well known that one of the applications of spherical harmonics to convexity is to the so called uniqueness results, and also to stability results. In this paper, we consider sections and half volumes $V(K\cap u^+)$ of star body K, where $u^+=\{x : x\in\mathbb{R}^d, x\cdot u\geqslant 0\}$. Using spherical harmonics, we show that the star bodies K, L are identical if they have the same volumes of their central sections and half volumes and we also prove a stability version of this result.

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