

MAXIMAL OPERATORS OF T MEANS WITH RESPECT TO WALSH–KACZMARZ SYSTEM

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Abstract. In this paper we prove and discuss some new $(H_p, L_{p,\infty})$ type inequalities of the maximal operators of T means with monotone coefficients with respect to Walsh-Kaczmarz system. It is also proved that these results are the best possible in a special sense. As applications, both some well-known and new results are pointed out. In particular, we apply these results to prove a.e. convergence of such T means.

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