

BESOV–MORREY SPACES AND VOLTERRA INTEGRAL OPERATOR

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Abstract. In this paper, we introduce a class of Besov-Morrey spaces $B_p^\lambda(s)$. For any positive Borel measure μ , we characterize the boundedness and compactness of the identity operator from $B_p^\lambda(s)$ spaces into tent spaces $T_l^q(\mu)$. As an application, the boundedness, compactness and essential norm of the Volterra integral operator T_g from $B_p^\lambda(s)$ spaces to some general function spaces are also investigated.

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REFERENCES

- [1] A. ALEMAN AND A. SISKAKIS, *An integral operator on H^p* , Complex Var. Theory Appl. **28** (1995), 149–158.
- [2] A. ALEMAN AND A. SISKAKIS, *Integration operators on Bergman spaces*, Indiana Univ. Math. J. **46** (1997), 337–356.
- [3] D. BLASI AND J. PAU, *A characterization of Besov type spaces and applications to Hankel type operators*, Michigan Math. J. **56** (2008), 401–417.
- [4] P. GALANOPOULOS, N. MERCHÁN AND A. SISKAKIS, *A family of Dirichlet-Morrey spaces*, Complex Var. Elliptic Equ. **64** (2019), 1686–1702.
- [5] P. GALINDO, M. LINDSTRÖM AND S. STEVIĆ, *Essential norm of operators into weighted-type spaces on the unit ball*, Abstr. Appl. Anal. Vol. 2011, Article ID 939873, (2011), 13 pages.
- [6] D. GIRELA AND J. PELÁEZ, *Carleson measure, multipliers and integration operators for spaces of Dirichlet type*, J. Funct. Anal. **241** (2006), 334–358.
- [7] B. HU AND S. LI, *$N(p, q, s)$ -type spaces in the unit ball of $\mathbb{C}^n(V)$: Riemann-Stieltjes operators and multipliers*, Bull. Sci. Math. **166** (2021), 102929, 27 pp.
- [8] L. HU, R. YANG AND S. LI, *Dirichlet-Morrey type spaces and Volterra integral operators*, J. Nonlinear Var. Anal. **5** (2021), 477–491.
- [9] P. LI, J. LIU AND Z. LOU, *Integral operators on analytic Morrey spaces*, Sci. China Math. **57** (2014), 1961–1974.
- [10] S. LI, J. LIU AND C. YUAN, *Embedding theorem for Dirichlet type spaces*, Canad. Math. Bull. **63** (2020), 106–117.
- [11] S. LI AND S. STEVIĆ, *Generalized weighted composition operators from α -Bloch spaces into weighted-type spaces*, J. Inequal. Appl. Vol. 2015, Article No. 265, (2015), 12 pages.
- [12] Q. LIN, J. LIU AND Y. WU, *Volterra type operators on $S^p(\mathbb{D})$ spaces*, J. Math. Anal. Appl. **461** (2018), 1100–1114.
- [13] X. LIU, S. LI AND R. QIAN, *Volterra integral operators and Carleson embedding on Campanato spaces*, J. Nonlinear Var. Anal. **5** (2021), 141–153.
- [14] J. LIU AND Z. LOU, *Carleson measure for analytic Morrey spaces*, Nonlinear Anal. **125** (2015), 423–432.
- [15] J. LIU, Z. LOU AND C. XIONG, *Essential norms of integral operators on spaces of analytic functions*, Nonlin. Anal. **75** (2012), 5145–5156.
- [16] J. PAU AND R. ZHAO, *Carleson measures, Riemann-Stieltjes and multiplication operators on a general family of function spaces*, Integral Equations Operator Theory **78** (2014), 483–514.

- [17] C. POMMERENKE, *Schlichte Funktionen und analytische Funktionen von beschränkter mittlerer Oszillation*, Comment. Math. Helv. **52** (1997), 591–602.
- [18] R. QIAN AND S. LI, *Volterra type operators on Morrey type spaces*, Math. Inequal. Appl. **18** (2015), 1589–1599.
- [19] R. QIAN AND S. LI, *Carleson measure and Volterra type operators on weighted BMOA spaces*, Georgian Math. J. **27** (2020), 413–424.
- [20] R. QIAN AND X. ZHU, *Embedding of Q_p spaces into tent spaces and Volterra integral operator*, AIMS Math. **6** (1) (2020), 698–711.
- [21] C. SHEN, Z. LOU AND S. LI, *Embedding of $BMOA_{\log}$ into tent spaces and Volterra integral operators*, Comput. Methods Funct. Theory. (2020), 1–18.
- [22] C. SHEN, Z. LOU AND S. LI, *Volterra integral operators from D_{p-2+s}^p into $F(p\lambda, p\lambda + s\lambda - 2, q)$* , Math. Inequal. Appl. **23** (2020), 1087–1103.
- [23] B. SEHBA AND S. STEVIĆ, *On some product-type operators from Hardy-Orlicz and Bergman-Orlicz spaces to weighted-type spaces*, Appl. Math. Comput. **233** (2014), 565–581.
- [24] Y. SHI AND S. LI, *Essential norm of integral operators on Morrey type spaces*, Math. Inequal. Appl. **19** (2016), 385–393.
- [25] A. SISKAKIS AND R. ZHAO, *A Volterra type operator on spaces of analytic functions*, Contemp. Math. **232** (1999), 299–312.
- [26] S. STEVIĆ, *On a new integral-type operator from the Bloch space to Bloch-type spaces on the unit ball*, J. Math. Anal. Appl. **354** (2009), 426–434.
- [27] S. STEVIĆ, *Norm and essential norm of an integral-type operator from the Dirichlet space to the Bloch-type space on the unit ball*, Abstr. Appl. Anal. Vol. 2010, Article ID 134969, (2010), 9 pages.
- [28] S. STEVIĆ, *Essential norm of some extensions of the generalized composition operators between k th weighted-type spaces*, J. Inequal. Appl. Vol. 2017, Article No. 220, (2017), 13 pages.
- [29] S. STEVIĆ AND Z. JIANG, *Boundedness and essential norm of an integral-type operator on a Hilbert-Bergman-type spaces*, J. Inequal. Appl. Vol. 2019, Article No. 121, (2019), 27 pages.
- [30] S. STEVIĆ, A. SHARMA AND A. BHAT, *Essential norm of products of multiplication composition and differentiation operators on weighted Bergman spaces*, Appl. Math. Comput. **218** (2011), 2386–2397.
- [31] S. STEVIĆ, A. SHARMA AND A. BHAT, *Products of multiplication composition and differentiation operators on weighted Bergman spaces*, Appl. Math. Comput. **217** (2011), 8115–8125.
- [32] M. TJANI, *Compact composition operators on some Möbius invariant Banach spaces*, Michigan State University, Department of Mathematics (1996).
- [33] Z. WU AND C. XIE, *Q spaces and Morrey spaces*, J. Funct. Anal. **201** (2003), 282–297.
- [34] R. ZHAO, *On a general family of function spaces*, Ann. Acad. Sci. Fenn. Math. Diss. **105** (1996), 56.
- [35] J. ZHOU AND X. ZHU, *Essential norm of a Volterra-type integral operator from Hardy spaces to some analytic function spaces*, J. Integral Equations Appl. **28** (2016), 581–593.
- [36] K. ZHU, *Operator theory in function spaces*, 2nd edn, American Mathematical Society, Providence (2007).