

ON THE HARDY PROPERTY OF MIXED MEANS

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Abstract. Hardy property of means has been extensively studied by Páles and Pasteczka since 2016. The core of this research is based on few of their properties: concavity, symmetry, monotonicity, repetition invariance and homogeneity (last axiom was recently omitted using some homogenizations techniques). In the present paper we deliver a study of possible omitting monotonicity and replacing repetition invariance by a weaker axiom.

These results are then used to establish the Hardy constant for certain types of mixed means.

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