QUANTITATIVE WEIGHTED ESTIMATES AND WEIGHTED COMPACTNESS FOR VARIATION OF APPROXIMATE IDENTITIES

YONGMING WEN, QUANQING FANG AND XIANMING HOU *

Abstract. In this paper, we give the quantitative weighted *BMO* estimates and C_q estimates for variation of approximate identities. Meanwhile, we also give a new characterization of $CMO(\mathbb{R}^n)$ via the compactness of the variation operators associated with commutators of approximate identities in weighted Lebesgue spaces.

Mathematics subject classification (2020): 42B25, 42B35, 47B47.
Keywords and phrases: Variation, endpoint estimate, C_q estimate, commutators, compactness.

REFERENCES

[1] J. J. BETANCOR, J. C. FARINA, E. HARBOUR AND L. RODRIGUEZ-MESA, *L^p-boundedness properities of variation operators in the Schrödinger setting*, Rev. Mat. Complut., **26**, 2 (2013), 485–534.

[2] S. BLOOM, *A commutator theorem and weighted BMO*, Trans. Amer. Math. Soc., **292**, 1 (1985), 103–122.

[3] J. BOURGAIN, *Pointwise ergodic theorems for arithmetic sets*, Publ. Math. Inst. Hautes Études Sci., **69**, 1 (1989), 5–45.

[4] T. A. BUI, *Boundedness of variation operators and oscillation operators for certain semigroups*, Nonlinear Anal., **106** (2014), 124–137.

[5] J. T. CAMPBELL, R. L. JONES, K. REINHOLD AND M. WIERDL, *Oscillations and variation for the Hilbert transform*, Duke Math. J., **105**, 1 (2000), 59–83.

[6] J. T. CAMPBELL, R. L. JONES, K. REINHOLD AND M. WIERDL, *Oscillations and variation for singular integrals in higher dimensions*, Trans. Amer. Math. Soc., **355**, 5 (2003), 2115–2137.

[7] J. CANTO, K. LI, L. RONCAL AND O. TAPIOLA, *C_p estimates for rough homogeneous singular integrals and sparse forms*, arXiv: 1909.08344v1.

[8] J. CANTO AND C. PÉREZ, *Extensions of the John-Nirenberg theorem and applicatons*, Proc. Amer. Math. Soc., **149**, 4 (2021), 1507–1525.

[9] J. CHEN AND G. HU, *Compact commutators of rough singular integral operators*, Canad. Math. Bull., **58**, 1 (2015), 19–29.

[10] Y. CHEN AND Y. DING, *Compactness characerization of commutators for Littlewood-Paley operators*, Kodai. Math. J., **32**, 2 (2009), 256–323.

[11] Y. CHEN, Y. DING, G. HONG AND H. LIU, *Weighted jump and variational inequalities for rough operators*, J. Funct. Anal., **275**, 8 (2018), 2446–2475.

[12] Y. CHEN, Y. DING, G. HONG AND H. LIU, *Variational inequalities for the commutators of rough operators with BMO functions*, arXiv: 1709.03125v1.

[13] Y. CHEN, Y. DING AND X. WANG, *Compactness of commutators for singular integrals on Morrey spaces*, Canad. J. Math., **64**, 2 (2012), 257–281.

[14] R. CHILL AND A. FIORENZA, *Singular integral operators with operator-valued kernels, and extrapolation of maximal regularity into rearrangement invariant Banach function spaces*, J. Evol. Equ., **14**, 4–5 (2014), 795–828.

[15] R. R. COIFMAN AND C. FEFFERMAN, *Weighted norm inequalities for maximal functions and singular integrals*, Studia Math., **51**, (1974), 241–250.

[16] R. R. COIFMAN, R. ROCHBERG AND G. WEISS, *Factorization theorems for Hardy spaces in several variables*, Ann. of Math. (2), **103**, 3 (1976), 611–635.

[17] R. CRESCIMBENI, F. J. MARTÍN-REYES, A. L. TORRE AND J. L. TORREA, *The ρ -variation of the Hermitian Riesz transform*, Acta Math. Sin. (Engl. Ser.) **26**, 10 (2010), 1827–1838.

[18] R. CRESCIMBENI, R. A. MACÍAS, T. MENÁRGUEZ, J. L. TORREA AND B. VIVIANI, *The ρ -variation as an operator between maximal operators and singular integrals*, J. Evol. Equ., **9**, 1 (2009), 81–102.

[19] A. CRIADO, C. PÉREZ AND I. P. RIVERA-RÍOS, *Sharp quantitative weighted BMO estimates and a new proof of the Harboure-Macías-Segovia's extrapolation theorem*, New Trends in Applied Harmonic Analysis, Volume 2. Birkhäuser, Cham (2019): 241–256.

[20] Y. DING, G. HONG AND H. LIU, *Jump and variational inequalities for rough operators*, J. Fourier. Anal. Appl., **23**, 3 (2017), 679–711.

[21] Y. DING, T. MEI AND Q. XUE, *Compactness of maximal commutators of bilinear, Calderón-Zygmund singular integral operators*, Some topics in harmonic analysis and applications, 163–175, Adv. Lect. Math. (ALM), 34, Int. Press, Somerville, MA, 2016.

[22] T. A. GILLESPIE AND J. L. TORREA, *Dimension free estimates for the oscillation of Riesz transform*, Israel J. Math., **141**, (2004), 125–144.

[23] L. GRAFAKOS, *Modern Fourier Analysis*, volume 250 of Graduate Texts in Mathematics, Springer, New York, Third edition, 2014.

[24] W. GUO, Y. WEN, H. WU AND D. YANG, *On the compactness of oscillation and variation of commutators*, Banach J. Math. Anal., (2021), `https://doi.org/10.1007/s43037-021-00123-z`.

[25] W. GUO, Y. WEN, H. WU AND D. YANG, *Variational characterizations of $H^p(\omega)$ and $BMO_\omega(\mathbb{R}^n)$*, arXiv:2010.00765v1.

[26] W. GUO, H. WU AND D. YANG, *A revisit on the compactness of commutators*, Canad. J. Math., (2020), `https://doi.org/10.4153/S0008414X20000644`.

[27] W. GUO AND G. ZHAO, *On relatively compact sets in quasi-Banach function spaces*, Proc. Amer. Math. Soc., **148**, 8 (2020), 3359–3373.

[28] E. HARBOURE, R. A. MACÍAS AND C. SEGOVIA, *Extrapolation results for classes of weights*, Amer. J. Math., **110**, 3 (1988), 383–397.

[29] T. P. HYTÖNEN, M. T. LACEY AND C. PÉREZ, *Sharp weighted bounds for the q-variation of singular integrals*, Bull. London Math. Soc., **45**, 3 (2013), 529–540.

[30] T. P. HYTÖNEN AND C. PÉREZ, *Sharp weighted bounds involving A_∞*, Anal. PDE, **6**, 4 (2013), 777–818.

[31] R. L. JONES, *Ergodic theory and connections with analysis and probability*, New York J. Math., 3A (1997), 31–67.

[32] R. L. JONES, *Variation inequalities for singular integrals and related operators*, Contemp. Math., **411**, (2006), 89–122.

[33] R. L. JONES AND K. REINHOLD, *Oscillation and variation inequalities for convolution powers*, Ergodic Theory Dynam. Systems., **21**, 6 (2001), 1809–1829.

[34] D. LÉPINGLE, *La variation d'ordre p des semi-martingales*, Z. Wahrscheinlichkeitstheorie und Verw. Gebiete, **36**, 4 (1976), 295–316.

[35] A. K. LERNER, *A note on the Coifman-Fefferman and Fefferman-Stein inequalities*, Ark. Mat., **58**, 2 (2020), 357–367.

[36] K. LI, C. PÉREZ, I. P. RIVERA-RÍOS AND L. RONCAL, *Weighted norm inequalities for rough singular integral operators*, J. Geom. Anal., **29**, 3 (2019), 2526–2564.

[37] F. LIU AND H. WU, *A criterion on oscillation and variation for the commutators of singular integrals*, Forum Math., **27** (2015), 77–97.

[38] H. LIU, *Variational characterization of H^p*, Proc. Roy. Soc. Edinburgh Sect. A, **149**, 5 (2019), 1123–1134.

[39] T. MA, J. L. TORREA AND Q. XU, *Weighted variation inequalities for differential operators and singular integrals in higher dimensions*, Sci. China Math., **60**, 8 (2017), 1419–1442.

[40] B. MUCKENHOUPT, *Norm inequalities relating the Hilbert transform to the Hardy-Littlewood maximal function*, Trans. Amer. Math. Soc., **165**, (1972), 207–226.

[41] B. MUCKENHOUPT AND R. WHEEDEN, *Weighted bounded mean oscillation and the Hilbert transform*, Studia Math., **54**, 3 (1976), 221–237.

[42] J. L. RUBIO DE FRANCIA, *Factorization theory and A_p weights*, Amer. J. Math., **106**, 3 (1984), 533–547.

[43] E. SAWYER, *Norm inequalities relating singular integrals and the maximal function*, Studia Math., **75**, 3 (1983), 253–263.

[44] A. UCHIYAMA, *On the compactness of operators of Hankel type*, Tôhoku Math. J. (2), **30**, 1 (1978), 163–171.

[45] H. WU AND D. YANG, *Charcterizations of weighted compactness of commutators via* $\mathrm{CMO}(\mathbb{R}^n)$, Proc. Amer. Math. Soc., **146**, 10 (2018), 4239–4254.

[46] K. YABUTA, *Sharp maximal function and* C_p *condition*, Arch. Math., **55**, (1990), 151–155.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com