SOME HARDY–TYPE INEQUALITIES IN BANACH FUNCTION SPACES

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Abstract. Some new inequalities of Hardy-type in Banach function space settings are proved and discussed. In particular, these results generalize and unify several classical Hardy-type inequalities. Some results are new also in the classical situation.

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