Mathematical
Inequalities
& Applications
Volume 24, Number 4 (2021), 1017–1030
doi:10.7153/mia-2021-24-71

NUMERICAL RADIUS IN HILBERT C^*–MODULES

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Abstract. Utilizing the linking algebra of a Hilbert C^*-module $\left(\mathscr{V}, \|\cdot\|\right)$, we introduce $\Omega(x)$ as a definition of numerical radius for an element $x \in \mathscr{V}$ and then show that $\Omega(\cdot)$ is a norm on $\mathscr{V}$ such that $\frac{1}{2}\|x\| \leqslant \Omega(x) \leqslant \|x\|$. In addition, we obtain an equivalent condition for $\Omega(x) = \frac{1}{2}\|x\|$. Moreover, we present a refinement of the triangle inequality for the norm $\Omega(\cdot)$. Some other related results are also discussed.

Mathematics subject classification (2020): 46L05, 47A30, 47A12, 46B20.
Keywords and phrases: C^*-algebra, Hilbert C^*-module, linking algebra, numerical range, numerical radius, inequality.

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