

ORLICZ DUAL LOGARITHMIC MINKOWSKI INEQUALITY

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Abstract. In this paper, we establish an Orlicz dual logarithmic Minkowski inequality by introducing a new concept of Orlicz dual mixed volume measure, and using the newly established Orlicz dual Minkowski inequality. The Orlicz dual logarithmic Minkowski inequality in special case yields the dual logarithmic Minkowski inequality. The L_p -dual mixed volume measure and L_p -dual logarithmic Minkowski inequality are first derived here.

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