SOME NOTES ON JENSEN–MERCER'S TYPE INEQUALITIES; EXTENSIONS AND REFINEMENTS WITH APPLICATIONS

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Abstract. In this paper we study inequalities corresponding to Jensen-Mercer's inequality. Some new extensions of Niezgoda's inequality and the integral version of Jensen-Mercer's inequality are given. The obtained inequalities do not only generalize the former ones, but our proofs are natural and simple. They clearly show the structure of such inequalities: they consist of two parts, a discrete or integral Jensen's inequality and then a majorization type inequality. Another purpose of the paper is to provide a deeper understanding of the methods used to refine Jensen-Mercer's and the corresponding inequalities. Moreover, some new refinements of these inequalities are obtained. Finally, some applications related to Fejér's and Hermite-Hadamard inequalities are given.

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