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REMARK ON THE CHAIN RULE OF FRACTIONAL DERIVATIVE IN THE SOBOLEV FRAMEWORK

KAZUMASA FUJIWARA

Abstract. A chain rule for power product is studied with fractional differential operators in the framework of Sobolev spaces. The fractional differential operators are defined by the Fourier multipliers. The chain rule is considered newly in the case where the order of differential operators is between one and two. The study is based on the analogy of the classical chain rule or Leibniz rule.

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