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MATRIX–VALUED POSITIVE DEFINITE KERNELS GIVEN BY EXPANSIONS: STRICT POSITIVE DEFINITENESS

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Abstract. Matrix functions of the form $(x,y) \in \Omega \times \Omega \mapsto \sum_\alpha A_\alpha f_\alpha(x,y)$, in which Ω is a nonempty set, the A_α are positive semi-definite matrices of the same fixed order, the f_α are complex-valued positive definite kernels on Ω, and the series is convergent for all x and y in Ω define matrix-valued positive definite kernels on Ω. Here, the sum may be multi-indexed, Ω may be endowed with either a topological or a metric structure, and $\{f_\alpha\}$ may inherit properties attached to the setting. In this paper, we present a criterion that establishes an abstract necessary and sufficient condition in order that the kernel is strictly positive definite on Ω. We point some implications and connections of the criterion in some relevant and concrete settings in order to motivate future work on the topic.

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