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HARNACK INEQUALITIES FOR FUNCTIONAL SDES DRIVEN BY SUBORDINATE MULTIFRACTIONAL BROWNIAN MOTION

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Abstract. Being base on the Girsanov theorem for multifractional Brownian motion, which can be constructed by the multifractional derivative operator, we establish the Harnack inequalities for a class of stochastic functional differential equations driven by subordinate multifractional Brownian motion by an approximation technique.

Mathematics subject classification (2020): 60H15, 60G15, 60H05.
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