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NORM OF THE DISCRETE CESÀRO OPERATOR MINUS IDENTITY

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Abstract. The norm of $C-I$ on ℓ^p, where C is the Cesàro operator, is shown to be $1/(p-1)$ when $1<p\leqslant 2$. This verifies a recent conjecture of G. J. O. Jameson. The norm of $C-I$ on ℓ^p is also determined when $2<p<\infty$. The two parts together answer a question raised by G. Bennett in 1996. Operator norms in the continuous case, Hardy's averaging operator minus identity, are already known. Norms in the discrete and continuous cases coincide.

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