RATE OF GROWTH OF DISTRIBUTIONALLY CHAOTIC FUNCTIONS

CLIFFORD GILMORE, FÉLIX MARTÍNEZ-GIMÉNEZ AND ALFRED PERIS *

Abstract. We investigate the permissible growth rates of functions that are distributionally chaotic with respect to differentiation operators. We improve on the known growth estimates for D-distributionally chaotic entire functions, where growth is in terms of average L^p-norms on spheres of radius $r > 0$ as $r \to \infty$, for $1 \leqslant p \leqslant \infty$. We compute growth estimates of $\partial/\partial x_k$-distributionally chaotic harmonic functions in terms of the average L^2-norm on spheres of radius $r > 0$ as $r \to \infty$. We also calculate sup-norm growth estimates of distributionally chaotic harmonic functions in the case of the partial differentiation operators D^α.

REFERENCES

[1] A. A. ALBANESE, X. BARRACHINA, E. M. MANGINO, AND A. PERIS, *Distributional chaos for strongly continuous semigroups of operators*, Commun. Pure Appl. Anal. **12**, 5 (2013), 2069–2082.

[2] M. P. ALDRED AND D. H. ARMITAGE, *Harmonic analogues of G. R. MacLane's universal functions*, J. London Math. Soc. (2) **57**, 1 (1998), 148–156.

[3] M. P. ALDRED AND D. H. ARMITAGE, *Harmonic analogues of G. R. Mac Lane's universal functions. II*, J. Math. Anal. Appl. **220**, 1 (1998), 382–395.

[4] D. H. ARMITAGE AND S. J. GARDINER, *Classical potential theory*, Springer Monographs in Mathematics, Springer-Verlag, London, 2001.

[5] S. AXLER, P. BOURDON, AND W. RAMEY, *Harmonic function theory, second edition*, volume 137 of Graduate Texts in Mathematics, Springer-Verlag, New York, 2001.

[6] F. BAYART AND S. GRIVAUX, *Frequently hypercyclic operators*, Trans. Amer. Math. Soc. **358**, 11 (2006), 5083–5117.

[7] F. BAYART AND É. MATHERON, *Dynamics of linear operators*, volume 179 of Cambridge Tracts in Mathematics, Cambridge University Press, Cambridge, 2009.

[8] F. BAYART AND I. Z. RUZSA, *Difference sets and frequently hypercyclic weighted shifts*, Ergodic Theory Dynam. Systems **35**, 3 (2015), 691–709.

[9] B. BEAUZAMY, *Introduction to operator theory and invariant subspaces*, volume 42 of North-Holland Mathematical Library, North-Holland Publishing Co., Amsterdam, 1988.

[10] T. BERMÚDEZ, A. BONILLA, F. MARTÍNEZ-GIMÉNEZ, AND A. PERIS, *Li-Yorke and distributionally chaotic operators*, J. Math. Anal. Appl. **373**, 1 (2011), 83–93.

[11] L. BERNAL-GONZÁLEZ AND A. BONILLA, *Order of growth of distributionally irregular entire functions for the differentiation operator*, Complex Var. Elliptic Equ. **61**, 8 (2016), 1176–1186.

[12] N. C. BERNARDES, A. BONILLA, AND A. PERIS, *Mean Li-Yorke chaos in Banach spaces*, J. Funct. Anal. **278**, 3 (2020), 108343, 31.

[13] N. C. BERNARDES, JR., A. BONILLA, V. MÜLLER, AND A. PERIS, *Distributional chaos for linear operators*, J. Funct. Anal. **265**, 9 (2013), 2143–2163.

[14] N. C. BERNARDES, JR., A. BONILLA, V. MÜLLER, AND A. PERIS, *Li-Yorke chaos in linear dynamics*, Ergodic Theory Dynam. Systems **35**, 6 (2015), 1723–1745.

[15] N. C. BERNARDES, JR., A. BONILLA, A. PERIS, AND X. WU, *Distributional chaos for operators on Banach spaces*, J. Math. Anal. Appl. **459**, 2 (2018), 797–821.

[16] O. BLASCO, A. BONILLA, AND K.-G. GROSSE-ERDMANN, *Rate of growth of frequently hypercyclic functions*, Proc. Edinb. Math. Soc. (2) **53**, 1 (2010), 39–59.

[17] J. BONET AND A. BONILLA, *Chaos of the differentiation operator on weighted Banach spaces of entire functions*, Complex Anal. Oper. Theory **7**, 1 (2013), 33–42.

[18] J. A. CONEJERO, M. KOSTIĆ, P. J. MIANA, AND M. MURILLO-ARCILA, *Distributionally chaotic families of operators on Fréchet spaces*, Commun. Pure Appl. Anal. **15**, 5 (2016), 1915–1939.

[19] D. DRASIN AND E. SAKSMAN, *Optimal growth of entire functions frequently hypercyclic for the differentiation operator*, J. Funct. Anal. **263**, 11 (2012), 3674–3688.

[20] S. M. DUĬOS-RUIS, *Universal functions and the structure of the space of entire functions*, Dokl. Akad. Nauk SSSR **279**, 4 (1984), 792–795.

[21] C. GILMORE, *Linear dynamical systems*, Irish Math. Soc. Bull. **86**, 1 (2020), 47–77.

[22] C. GILMORE, E. SAKSMAN, AND H.-O. TYLLI, *Optimal growth of harmonic functions frequently hypercyclic for the partial differentiation operator*, Proc. Roy. Soc. Edinburgh Sect. A **149**, 6 (2019), 1577–1594.

[23] K.-G. GROSSE-ERDMANN, *On the universal functions of G. R. MacLane*, Complex Variables Theory Appl. **15**, 3 (1990), 193–196.

[24] K.-G. GROSSE-ERDMANN AND A. PERIS MANGUILLOT, *Linear chaos*, Universitext, Springer, London, 2011.

[25] Y. KATZNELSON, *An introduction to harmonic analysis*, second edition, Dover Publications, New York, 1976.

[26] Ü. KURAN, *On Brelot-Choquet axial polynomials*, J. London Math. Soc. (2) **4**, 1 (1971), 15–26.

[27] T. Y. LI AND J. A. YORKE, *Period three implies chaos*, Amer. Math. Monthly **82**, 10 (1975), 985–992.

[28] G. R. MACLANE, *Sequences of derivatives and normal families*, J. Analyse Math. **2**, 1 (1952), 72–87.

[29] F. MARTÍNEZ-GIMÉNEZ, P. OPROCHA, AND A. PERIS, *Distributional chaos for backward shifts*, J. Math. Anal. Appl. **351**, 2 (2009), 607–615.

[30] F. MARTÍNEZ-GIMÉNEZ, P. OPROCHA, AND A. PERIS, *Distributional chaos for operators with full scrambled sets*, Math. Z. **274**, 1–2 (2013), 603–612.

[31] M. NIKULA, *Frequent hypercyclicity of random entire functions for the differentiation operator*, Complex Anal. Oper. Theory **8**, 7 (2014), 1455–1474.

[32] B. SCHWEIZER AND J. SMÍTAL, *Measures of chaos and a spectral decomposition of dynamical systems on the interval*, Trans. Amer. Math. Soc. **344**, 2 (1994), 737–754.

[33] S. A. SHKARIN, *On the growth of D-universal functions*, Moscow Univ. Math. Bull. **48**, 6 (1993), 49–51.

[34] X. WU AND G. CHEN, *Scrambled sets of shift operators*, J. Nonlinear Sci. Appl. **9**, 5 (2016), 2631–2637.

[35] X. WU, L. WANG, AND G. CHEN, *Weighted backward shift operators with invariant distributionally scrambled subsets*, Ann. Funct. Anal. **8**, 2 (2017), 199–210.

[36] X. WU AND P. ZHU, *Dense chaos and densely chaotic operators*, Tsukuba J. Math. **36**, 2 (2012), 367–375.

[37] Z. YIN AND Q. YANG, *Distributionally n-chaotic dynamics for linear operators*, Rev. Mat. Complut. **31**, 1 (2018), 111–129.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com