Mathematical
Inequalities
& Applications
Volume 25, Number 1 (2022), 169–186
doi:10.7153/mia-2022-25-11

APPROXIMATION BY LINEAR COMBINATIONS OF TRANSLATES OF A SINGLE FUNCTION

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Abstract. We study approximation of periodic functions by arbitrary linear combinations of n translates of a single function. We construct some linear methods of this approximation for univariate functions in the class induced by the convolution with a single function, and prove upper bounds of the L^p-approximation convergence rate by these methods, when $n \to \infty$, for $1 \leqslant p \leqslant \infty$. We also generalize these results to classes of multivariate functions defined as the convolution with the tensor product of a single function. In the case $p = 2$, for this class, we also prove a lower bound of the quantity characterizing best approximation of by arbitrary linear combinations of n translates of arbitrary function.

Mathematics subject classification (2020): 41A46, 41A63, 42A99.
Keywords and phrases: Function spaces induced by the convolution with a given function; Approximation by arbitrary linear combinations of n translates of a single function.

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