OPTIMAL CONSTANTS OF THE MIXED
LITTLEWOOD INEQUALITIES: THE COMPLEX CASE

Wasthenny Cavalcante, Daniel Núñez-Alarcón*,
Daniel Pellegrino and Pilar Rueda

Abstract. In this paper, among other results, we obtain an extension of a kind of Khinchine inequality given by R. Blei, namely, the Blei–Khinchine inequality. As an application we obtain the optimal constants of the mixed Littlewood inequalities, for complex scalars.

REFERENCES

[1] N. Albuquerque, F. Bayart, D. Pellegrino and J. B. Seoane-Sepúlveda, *Optimal Hardy–Littlewood type inequalities for polynomials and multilinear operators*, Israel J. Math. **211** (2016), no. 1, 197–220.

[2] G. Araújo, D. Pellegrino and D. Silva, *On the upper bounds for the constants of the Hardy–Littlewood inequality*, J. Funct. Anal. **264** (2014), 1878–1888.

[3] S. Arunachalam, S. Chakraborty, M. Koucký and N. Saurabh, R. de Wolf, *Improved bounds on Fourier entropy and Min-entropy*, (English summary) 37th International Symposium on Theoretical Aspects of Computer Science, Art. 45–19 pp., LIPIcs. Leibniz Int. Proc. Inform., 154, Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern, 2020.

[4] F. Bayart, *Multiple summing maps: coordinatewise summability, inclusion theorems and p-Sidon sets*, J. Funct. Anal. **274** (2018), no. 4, 1129–1154.

[5] F. Bayart, D. Pellegrino and J. Seoane-Sepulveda, *The Bohr radius of the n-dimensional polydisk is equivalent to $\sqrt{\frac{\log n}{n}}$*, Adv. Math. **264** (2014), 726–746.

[6] R. Blei, *Analysis in Integer and Fractional Dimensions*, Cambridge Studies in Advanced Mathematics, 71. Cambridge University Press, Cambridge, 2001. xx+556 pp.

[7] H. F. Bohnenblust and E. Hille, *On the absolute convergence of Dirichlet series*, Ann. of Math. (2) **32** (1931), 600–622.

[8] G. Botelho and D. Freitas, *Summing multilinear operators by blocks: The isotropic and anisotropic cases*, J. Math. Anal. Appl. **490** (2020), no. 1, 124203, 21 pp.

[9] W. Cavalcante and D. Núñez-Alarcón, *Remarks on the Hardy–Littlewood inequality for m-homogeneous polynomials and m-linear forms*, Quaest. Math. **39** (2016), no. 8, 1101–1113.

[10] J. Diestel, H. Jarchow and A. Tonge, *Absolutely summing operators*, Cambridge Studies in Advanced Mathematics, 43. Cambridge University Press, Cambridge, 1995. xvi+474 pp.

[11] V. Dimant and P. Sevilla-Peris, *Summation of coefficients of polynomials on ℓ_p spaces*, Publ. Mat. **60** (2016), no. 2, 289–310.

[12] D. J. H. Garling, *Inequalities: A Journey into Linear Analysis*, Cambridge University Press, 2007.

[13] U. Haagerup, *The best constants in the Khinchine inequality*, Studia Math. **70** (1982) 231–283.

[14] G. Hardy and J. E. Littlewood, *Bilinear forms bounded in space $[p,q]$*, Quart. J. Math. 5 (1934), 241–254.

[15] A. Khintchine, *Über dyadische Brüche*, (German) Math. Z. **18** (1923), no. 1, 109–116.

[16] H. König, *On the best constants in the Khintchine inequality for Steinhaus variables*, Israel J. Math. **203** (2014), 23–57

[17] J. E. LITTLEWOOD, *On bounded bilinear forms in an infinite number of variables*, The Quarterly Journal of Mathematics **1** (1930), 164–174.

[18] M. MAIA AND J. SANTOS, *On the mixed (ℓ_1,ℓ_2)-Littlewood inequalities and interpolation*, Math. Inequal. Appl. (3) **21**, (2018), 721–727.

[19] M. MASTYŁO AND E. A. SÁNCHEZ PÉREZ, *Factorization theorems for some new classes of multilinear operators*, Asian J. Math. **24** (2020), no. 1, 1–29.

[20] M. MASTYŁO AND E. B. SILVA, *Interpolation of compact bilinear operators*, Bull. Math. Sci. **10** (2020), no. 2, 2050002, 26 pp.

[21] A. MONTANARO, *Some applications of hypercontractive inequalities in quantum information theory*, J. Math. Phys. **53** (2012), no. 12, 122206, 15 pp.

[22] T. NOGUEIRA, D. NÚÑEZ-ALARCÓN AND D. PELLEGRINO, *Optimal constants for a mixed Littlewood type inequality*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **111** (2017), no. 4, 1083–1092.

[23] D. NÚÑEZ-ALARCÓN AND D. M. SERRANO-RODRÍGUEZ, *The best constants in the multiple Khintchine inequality*, Linear Multilinear Algebra **67** (2019), no. 11, 2325–2344.

[24] D. PELLEGRINO, *The optimal constants of the mixed (ℓ_1,ℓ_2)-Littlewood inequality*, J. Number Theory **160** (2016), 11–18.

[25] D. PELLEGRINO, J. SANTOS AND J. SEOANE-SEPULVEDA, *Some techniques on nonlinear analysis and applications*, Adv. Math. 229 (2012), no. 2, 1235–1265.

[26] D. PELLEGRINO AND E. TEIXEIRA, *Towards sharp Bohnenblust-Hille constants*, Commun. Contemp. Math. **20** (2018), no. 3, 1750029, 33 pp.

[27] T. PRACIANO-PEREIRA, *On bounded multilinear forms on a class of ℓ_p spaces*, J. Math. Anal. Appl. **81** (1981), 561–568.

[28] H. QUEFFÉLEC, *H. Bohr's vision of ordinary Dirichlet series: old and new results*, J. Anal. **3** (1995), 43–60.

[29] F. VIEIRA COSTA JUNIOR, *The optimal multilinear Bohnenblust-Hille constants: a computational solution for the real case*, Numer. Funct. Anal. Optim. **39** (2018), no. 15, 1656–1668.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com