SOME WEAK TYPE ESTIMATES FOR
MAXIMAL SINGULAR INTEGRALS

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Abstract. We consider some maximal singular integral operators having variable kernels on $\mathbb{R}^n$ with doubling measures and prove L^p and weak type estimates for them under certain conditions. Also, certain weighted weak type estimates are shown for maximal singular integrals with A_1 weights of Muckenhoupt for the Lebesgue measure.

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