

WEIGHTED INEQUALITIES FOR MULTILINEAR OPERATORS ACTING BETWEEN GENERALIZED ZYGMUND SPACES ASSUMING MUSIELAK–ORLICZ BUMPS CONDITIONS

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Abstract. We study continuity properties for multilinear operators between generalized Zygmund spaces of $L \log L$ type, in the variable exponent setting with different weights. In order to attack this goal we consider generalized bump conditions on the weights involved.

We shall be dealing with two different classes of operators. The former deals with operators dominated by multilinear sparse forms and the latter are potential operators and their commutators. These classes includes the multilinear Calderón-Zygmund operators, the bilinear Hilbert transform, the multilinear fractional integral operator and the multilinear Bessel potential, among others. The symbols of the commutators belong to some generalized spaces that include bounded mean oscillation spaces and the classical Lipschitz spaces.

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