

COVERING CONVEX HULLS OF COMPACT CONVEX SETS WITH SMALLER HOMOTHETIC COPIES

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Abstract. Estimations of the covering functional of a convex body that can be expressed as the convex hull of a finite number of compact convex sets are presented. In particular, an upper bound of the covering functional of zonotopes that can be written as the sum of $n+k$ segments is given.

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