ON BACKWARD ALUTHGE ITERATES
OF COMPLEX SYMMETRIC OPERATORS

EUNGIL KO, JI EUN LEE AND MEE-JUNG LEE*

Abstract. For a nonnegative integer k, an operator $T \in \mathcal{L}(\mathcal{H})$ is called a *backward Aluthge iterate of a complex symmetric operator of order* k if the kth Aluthge iterate $\widetilde{T}^{(k)}$ of T is a complex symmetric operator, denoted by $T \in BAIC(k)$. In this paper, we study several properties of the backward Aluthge iterate of a complex symmetric operator. We show that every nilpotent operator of order $k+2$ belongs to $BAIC(k)$. Moreover, we prove that if T belongs to $BAIC(k)$, then T has the property (β) if and only if T is decomposable. Finally, we show that, under some conditions, operators in $BAIC(k)$ have nontrivial hyperinvariant subspaces and we consider Weyl type theorems for such operators.

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