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ON THE ERDŐS–LAX AND TURÁN INEQUALITIES CONCERNING POLYNOMIALS

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Abstract. In this paper, we prove certain sharp inequalities that relate the uniform norm of the derivative and the polynomial itself, in case when the zeros are outside or inside some closed disk. We further extend the obtained results to the polar derivative of a polynomial. The obtained results strengthen some recently proved Erdős-Lax and Turán-type inequalities contained in a paper published recently by Kumar [*Complex Anal. Oper. Theory* **14**, 65 (2020)], as well as other related inequalities.

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