

DISTORTION IN THE METRIC CHARACTERIZATION OF SUPERREFLEXIVITY IN TERMS OF THE INFINITE BINARY TREE

SOFIYA OSTROVSKA

Abstract. The article presents a quantitative refinement of the result of Baudier (*Archiv Math.*, **89** (2007), no. 5, 419–429): the infinite binary tree admits a bilipschitz embedding into an arbitrary non-superreflexive Banach space. According to the results of this paper, we can additionally require that, for an arbitrary $\varepsilon > 0$ and an arbitrary non-superreflexive Banach space X , there is an embedding of the infinite binary tree into X whose distortion does not exceed $4 + \varepsilon$.

Mathematics subject classification (2020): 46B85, 46B20, 51F30.

Keywords and phrases: Distortion of a bilipschitz embedding, logarithmic spiral, superreflexive Banach space, test space.

REFERENCES

- [1] K. BALL, *The Ribe programme*, Séminaire Bourbaki, vol. 2011/2012, Exposés 1043–1058, Astérisque No. **352** (2013), Exp. No. 1047, viii, 147–159.
- [2] F. BAUDIER, *Metric characterization of super-reflexivity and linear type of Banach spaces*, *Archiv Math.*, **89**, 5 (2007), 419–429.
- [3] F. BAUDIER, *Embeddings of proper metric spaces into Banach spaces*, *Houston J. Math.*, **38**, 1 (2012), 209–223.
- [4] F. BAUDIER, G. LANCIEN, *Embeddings of locally finite metric spaces into Banach spaces*, *Proc. Amer. Math. Soc.*, **136**, (2008), 1029–1033.
- [5] B. BEAUZAMY, *Introduction to Banach spaces and their geometry*, North Holland Publishing Co., Amsterdam, 1982.
- [6] Y. BENYAMINI, J. LINDENSTRAUSS, *Geometric nonlinear functional analysis*, vol. **1**, American Mathematical Society Colloquium Publications, **48**, American Mathematical Society, Providence, RI, 2000.
- [7] J. BOURGAIN, *The metrical interpretation of superreflexivity in Banach spaces*, *Israel J. Math.*, **56**, 2 (1986), 222–230.
- [8] J. BOURGAIN, V. MILMAN, H. WOLFSON, *On type of metric spaces*, *Trans. Amer. Math. Soc.*, **294**, 1 (1986), 295–317.
- [9] A. BRUNEL, L. SUCHESTON, *On J -convexity and some ergodic super-properties of Banach spaces*, *Trans. Amer. Math. Soc.* **204**, (1975), 79–90.
- [10] S. BUYALO, V. SCHROEDER, *Elements of asymptotic geometry*, EMS Monographs in Mathematics, European Mathematical Society (EMS), Zürich, 2007.
- [11] R. C. JAMES, *Uniformly non-square Banach spaces*, *Annals of Math.*, **80**, (1964), 542–550.
- [12] R. C. JAMES, *Some self-dual properties of normed linear spaces*, in: *Symposium on Infinite-Dimensional Topology* (Louisiana State Univ., Baton Rouge, La., 1967), pp. 159–175. *Ann. of Math. Studies*, no. **69**, Princeton Univ. Press, Princeton, N.J., 1972.
- [13] J. LINDENSTRAUSS, L. TZAFRIRI, *Classical Banach spaces. I. Sequence spaces*. *Ergebnisse der Mathematik und ihrer Grenzgebiete*, vol. **92**, Springer-Verlag, Berlin-New York, 1977.
- [14] A. NAOR, *An introduction to the Ribe program*, *Jpn. J. Math.*, **7**, 2 (2012), 167–233.
- [15] A. NAOR, *Metric dimension reduction: a snapshot of the Ribe program*, *Proc. Int. Cong. of Math.* 2018, Rio de Janeiro, vol. **1**, 759–838.

- [16] S. OSTROVSKA, M. I. OSTROVSKII, *Distortion in the finite determination result for embeddings of locally finite metric spaces into Banach spaces*, *Glasg. Math. J.*, **61**, 1 (2019), 33–47.
- [17] M. I. OSTROVSKII, *Coarse embeddability into Banach spaces*, *Topology Proc.*, **33** (2009), 163–183.
- [18] M. I. OSTROVSKII, *Embeddability of locally finite metric spaces into Banach spaces is finitely determined*, *Proc. Amer. Math. Soc.*, **140**, (2012), 2721–2730.
- [19] M. I. OSTROVSKII, *Different forms of metric characterizations of classes of Banach spaces*, *Houston. J. Math.*, **39**, 3 (2013), 889–906.
- [20] M. I. OSTROVSKII, *Metric Embeddings: Bilipschitz and Coarse Embeddings into Banach Spaces*, de Gruyter Studies in Mathematics, **49**, Walter de Gruyter & Co., Berlin, 2013.
- [21] M. I. OSTROVSKII, *Metric characterizations of some classes of Banach spaces*, in: *Harmonic Analysis, Partial Differential Equations, Complex Analysis, Banach Spaces, and Operator Theory, Celebrating Cora Sadosky's life*, M. C. Pereyra, S. Marcantognini, A. M. Stokolos, W. U. Romero (Eds.), Association for Women in Mathematics Series, Vol. **4**, pp. 307–347, Springer-Verlag, Berlin, 2016.
- [22] G. PISIER, *Martingales in Banach spaces*, Cambridge Studies in Advanced Mathematics **155**, Cambridge, Press Cambridge University Press, 2016.
- [23] J. J. SCHÄFFER, K. SUNDARESAN, *Reflexivity and the girth of spheres*, *Math. Ann.*, **184** (1969/1970), 163–168.