

## ON HERMITE–HADAMARD INEQUALITIES FOR $(k, h)$ –CONVEX SET–VALUED MAPS

KAZIMIERZ NIKODEM AND TERESA RAJBA

*Abstract.* We introduce the class of  $(k, h)$ -convex set-valued maps defined on  $k$ -convex domains by

$$h(t)G(x_1) + h(1-t)G(x_2) \subset G(k(t)x_1 + k(1-t)x_2), \quad x_1, x_2 \in D, \quad t \in [0, 1],$$

and prove a Hermite-Hadamard-type theorem for such maps. Many other properties of  $(k, h)$ -convex set-valued maps are also presented.

*Mathematics subject classification (2020):* 26A51, 39B62, 26D15.

*Keywords and phrases:*  $(k, h)$ -convexity, set-valued maps, Aumann integral, Hermite-Hadamard-type inclusions.

### REFERENCES

- [1] J. P. AUBIN, H. FRANKOWSKA, *Set-Valued Analysis*, Birkhäuser, Boston-Basel-Berlin (1990).
- [2] R. J. AUMANN, *Integrals of set-valued functions*, J. Math. Anal. Appl., **12** (1965), 1–12.
- [3] M. BOMBARDELLI, S. VAROŠANEC, *Properties of  $h$ -convex functions related to the Hermite-Hadamard-Fejér inequalities*, Comput. Math. Appl., **58**, 9 (2009), 1869–1877.
- [4] B. MICHERDA, T. RAJBA, *On some Hermite-Hadamard-Fejér inequalities for  $(k, h)$ -convex functions*, Math. Inequal. Appl., **15**, 4 (2012), 931–940.
- [5] F. C. MITROI, K. NIKODEM, SZ. WĄSOWICZ, *Hermite-Hadamard inequalities for convex set-valued functions*, Demonstr. Math., **46** (2013), 655–662.
- [6] K. NIKODEM,  *$K$ -convex and  $K$ -concave set-valued functions*, Zeszyty Nauk. Politech. Łódź. Mat. 559 (Rozprawy Nauk. 114), Łódź (1989), 1–75.
- [7] T. RAJBA, *On the Ohlin lemma for Hermite-Hadamard-Fejér type inequalities*, Math. Ineq. Appl., **17**, 557–571 (2014).
- [8] S. ROLEWICZ, *Functional Analysis and Control Theory. Linear Systems*, PWN – Polish Scientific Publishers & D. Reidel Publishing Company, Dordrecht/Boston/Lancaster/Tokyo (1987).
- [9] E. SADOWSKA, *Hadamard inequality and a refinement of Jensen inequality for set-valued functions*, Results Math., **32** (1997), 332–337.
- [10] S. VAROŠANEC, *On  $h$ -convexity*, J. Math. Anal. Appl., **326**, 1 (2007), 303–311.