ABSTRACT HARDY SPACES

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Abstract. In this paper, we study the abstract Hardy spaces on spaces of homogeneous X. Firstly, we give the definitions of the atomic Hardy spaces H^p_{ato} and the molecular Hardy spaces $H^p_{\varepsilon,mol}(0 < p < 1)$. Secondly, we give out the comparison between our Hardy spaces with some other Hardy spaces. Finally, we prove the continuity theorem of the sublinear operator on the Hardy spaces and give an example.

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