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ON REARRANGEMENT INEQUALITIES FOR MULTIPLE SEQUENCES

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Abstract. The classical rearrangement inequality provides bounds for the sum of products of two sequences under permutations of terms and show that similarly ordered sequences provide the largest value whereas opposite ordered sequences provide the smallest value. This has been generalized to multiple sequences to show that similarly ordered sequences provide the largest value. However, the permutations of the sequences that result in the smallest value are generally not known. We show a variant of the rearrangement inequality for which a lower bound can be obtained and conditions for which this bound is achieved for a sequence of permutations. We also study a generalization of the rearrangement inequality and a variation where the permutations of terms can be across the various sequences. For this variation, we can also find the minimizing and maximizing sequences under certain conditions. Finally, we also look at rearrangement inequalities of other objects that can be ordered such as functions and matrices.

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