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DISPERSION ESTIMATES FOR THE BOUNDARY INTEGRAL OPERATOR ASSOCIATED WITH THE FOURTH ORDER SCHRÖDINGER EQUATION POSED ON THE HALF LINE

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Abstract. In this paper, we prove dispersion estimates for the boundary integral operator associated with the fourth order Schrödinger equation posed on the half line. Proofs of such estimates for domains with boundaries are rare and generally require highly technical approaches, as opposed to our simple treatment which is based on constructing a boundary integral operator of oscillatory nature via the Fokas method. Our method is uniform and can be extended to other higher order partial differential equations where the main equation possibly involves more than one spatial derivatives.

Mathematics subject classification (2020): 35A22, 35A23, 35B65, 35C15, 35G16.
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