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REGULARITY OF COMMUTATOR OF BILINEAR MAXIMAL OPERATOR WITH LIPSCHITZ SYMBOLS

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Abstract. This paper is devoted to exploring the regularity properties of the commutator of maximal operator in the bilinear setting. More precisely, we introduce the commutator of bilinear maximal operator and bilinear maximal commutator. We establish some new boundedness and continuity for the above operators on the Sobolev spaces, Triebel–Lizorkin spaces and Besov spaces under the condition that the symbol function belongs to the Lipschitz space.

Mathematics subject classification (2020): 42B25, 46E35.
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