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# $q$–HERMITE—HADAMARD INEQUALITIES FOR FUNCTIONS WITH CONVEX OR $h$–CONVEX $q$–DERIVATIVE

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*Abstract.* In this work, using the definitions of convex functions and $h$-convex functions, new Hermite–Hadamard type inequalities are presented using the framework of $q$-calculus. We prove inequalities for the $q_a$- and $q^b$-definite integrals of functions which have a convex or general convex $q_a$- or $q^b$-derivative. These inequalities have consequences for $q$-integrals and classical integrals, while extending some results previously known from the literature.

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