

REFINEMENTS OF TWO DETERMINANTAL INEQUALITIES FOR POSITIVE SEMIDEFINITE MATRICES

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Abstract. Let $A, B, C \in \mathbb{C}^{n \times n}$ be positive semidefinite matrices and let $|A|, |B|, |C|$ be determinants of $A, B, C \in \mathbb{C}^{n \times n}$ respectively. In this paper, the authors prove two determinantal inequalities

$$|A+B+C|+|C| \geq |A+C|+|B+C|+(2^n-2)|AB|^{1/2}+3(3^{n-1}-2^n+1)|ABC|^{1/3}$$

and

$$|A+B+C|+|A|+|B|+|C| \geq |A+B|+|A+C|+|B+C|+3(3^{n-1}-2^n+1)|ABC|^{1/3}.$$

These two inequalities refine known ones.

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