

LOGARITHMICALLY COMPLETE MONOTONICITY OF A MATRIX-PARAMETRIZED ANALOGUE OF THE MULTINOMIAL DISTRIBUTION

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Abstract. In the paper, the authors introduce a matrix-parametrized generalization of the multinomial probability mass function that involves a ratio of several multivariate gamma functions. They show the logarithmically complete monotonicity of this generalization and derive several inequalities involving ratios of multivariate gamma functions.

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