

## FRACTIONAL INTEGRAL OPERATOR WITH ROUGH KERNEL ON VARIOUS CLOSED SUBSPACES OF MORREY SPACES

DANIEL SALIM\*, DENNY IVANAL HAKIM, YUDI SOEHARYADI  
AND WONO SETYA BUDHI

*Abstract.* In this paper we investigate fractional integral operators with rough kernel on certain closed subspaces of Morrey spaces. We prove that the operator maps vanishing Morrey spaces into themselves. In addition, we discuss the behavior of this operator on the closure in Morrey spaces of essentially bounded functions, compactly supported functions, and essentially bounded and compactly supported functions.

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