

AN APPLICATION OF THE WEIGHTED DISCRETE HARDY INEQUALITY

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Abstract. In this work, we use a well-known characterization of a weighted version of the classical discrete Hardy inequality to exhibit a sufficient condition for the existence of solutions of the differential equation $\operatorname{div} \mathbf{u} = f$ in weighted Sobolev spaces over a certain irregular planar domain. The solvability of this equation is fundamental for the analysis of the Stokes equations.

The proof follows from a local-to-global argument based on a certain decomposition of functions which is also of interest for its applications to other inequalities or related results in Sobolev spaces, such as the Korn inequality.

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