

NECESSARY AND SUFFICIENT CONDITIONS FOR BOUNDEDNESS OF COMMUTATORS OF BILINEAR HARDY–LITTLEWOOD MAXIMAL FUNCTION

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Abstract. Let \mathcal{M} be the bilinear Hardy-Littlewood maximal function and $\vec{b} = (b, b)$ be a collection of locally integrable functions. In this paper, the authors establish characterizations of the weighted BMO space in terms of several different commutators of bilinear Hardy-Littlewood maximal function, respectively; these commutators include the maximal iterated commutator $\mathcal{M}_{\Pi\vec{b}}$, the maximal linear commutator $\mathcal{M}_{\Sigma\vec{b}}$, the iterated commutator $[\Pi\vec{b}, \mathcal{M}]$ and the linear commutator $[\Sigma\vec{b}, \mathcal{M}]$.

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