HARDY OPERATORS AND COMMUTATORS ON GENERALIZED CENTRAL FUNCTION SPACES

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Abstract. In this paper, we would like to study the boundedness of operators of Hardy type on generalized central function spaces, such as the generalized central Hardy space $\mathbf{HA}_\varphi^p(\mathbb{R}^n)$, the generalized central Morrey space $\dot{\mathbf{M}}_\varphi^p(\mathbb{R}^n)$, and the generalized central Campanato space $\dot{\mathrm{CMO}}_\varphi^p(\mathbb{R}^n)$, with $p \in (1,\infty)$, and $\varphi(t) : (0,\infty) \to (0,\infty)$. We first show that $\mathbf{HA}_\varphi^{p'}(\mathbb{R}^n)$ is the predual of $\dot{\mathrm{CMO}}_\varphi^p(\mathbb{R}^n)$. After that, we investigate the boundedness of operators of Hardy type on those spaces. By duality, we obtain the boundedness characterization of function $b \in \dot{\mathrm{CMO}}_\varphi^p(\mathbb{R}^n)$ via the $\dot{\mathbf{M}}_\varphi^p(\mathbb{R}^n)$-boundedness of commutator $[b, \mathscr{H}^*]$.

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