

RUBIO DE FRANCIA EXTRAPOLATION RESULTS FOR GRAND LEBESGUE SPACES DEFINED ON SETS HAVING POSSIBLY INFINITE MEASURE

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Abstract. In this paper, we present Rubio de Francia type extrapolation results for certain generalized grand Lebesgue spaces defined on sets $\Omega \subseteq \mathbb{R}^n$ with $|\Omega| \leq \infty$. Both diagonal and off-diagonal cases have been considered. As applications to these results, boundedness of certain integral operators has been studied and also a vector valued inequality has been established.

Mathematics subject classification (2020): 46E35, 26D10.

Keywords and phrases: Rubio de Francia extrapolation, grand Lebesgue space, Muckenhoupt class of weights, Muckenhoupt-Wheeden class of weights, integral operators.

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