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ON THE SUMMABILITY OF A CLASS OF FORMAL POWER SERIES

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Abstract. The formal power series solutions for some classes of moment differential equations, induced by polynomial moment differential operators, are characterized in terms of their summability properties, and in terms of estimates for recursive expressions involving their coefficients. Of special interest are the particularization of these results to classes of fractional and of ordinary differential equations. The Stokes' phenomenon can be described in some of these situations. The main results are extended into the framework of q-Gevrey asymptotics and q-difference equations.

Mathematics subject classification (2020): 34A25, 34A08, 34K25, 40C10.

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REFERENCES

[1] W. BALSER, *Formal power series and linear systems of meromorphic ordinary differential equations*, Springer, Berlin, 2000.
[2] W. BALSER, M. YOSHINO, *Gevrey order of formal power series solutions of inhomogeneous partial differential equations with constant coeffcients*, Funkcialaj Ekvac. 53 (2010), 411–434.
[3] B. L. J. BRAAKSMA, *Transseries for a class of nonlinear differerence equations*, J. Difference Equ. Appl. 7 (5) (2001), 717–750.
[4] B. L. J. BRAAKSMA, *Multisummability of formal power series solutions of nonlinear meromorphic differential equations*, Ann. Inst. Fourier (Grenoble) 42 (1992), 517–540.
[5] O. COSTIN, *Asymptotics and Borel Summability*, CRC Press, 2008.
[6] O. COSTIN, X. XIA, *From the Taylor series of analytic functions to their global analysis*, Nonlinear Anal. 119 (2015), 106–114.
[7] J. ECALLE, *Les fonctions résurgentes I, II*, Publ. Math. Orsay, Univ. Paris XI, Orsay, 1981; III, Publ. Math. Orsay, Univ. Paris XI, Orsay, 1985.
[8] J. ECALLE, *Introduction à l'Accélération et à ses Applications*, Travaux en Cours, Hermann, Paris, 1993.
[9] J. ECALLE, *Bifurcations and periodic orbits of vector fields*, in: NATO ASI Series, vol. 408, 1993.
[10] A. A. GOLDBERG, I. V. OSTROVSKII, *Value distribution of meromorphic functions*, Transl. Math. Monogr. 236, Amer. Math. Soc., Providence, RI, 2008.
[11] W. A. GROENING, *Quasi-analyticity for functions of several variables*, Duke Math. J. 38 (1971), 109–115.
[12] J. JIMÉNEZ-GARRIDO, J. SANZ, G. SCHINDL, *Log-convex sequences and nonzero proximate orders*, J. Math. Anal. Appl. 448 (2017), no. 2, 1572–1599.
[13] J. JIMÉNEZ-GARRIDO, J. SANZ, G. SCHINDL, *Injectivity and surjectivity of the asymptotic Borel map in Carleman ultraholomorphic classes*, J. Math. Anal. Appl. 469 (2019), no. 1, 136–168.
[14] A. KILBAS, H. SRIVASTAVA, J. TRUJILLO, *Theory and applications of fractional differential equations*, North-Holland Math. Stud. 204, Elsevier, Amsterdam (2006).
[15] H. KOMATSU, *Ultradistributions, I: Structure theorems and a characterization*, J. Fac. Sci. Univ. Tokyo Sect. IA Math. 20 (1973), 25–105.
[16] A. LASTRA, S. MALEK, *On q-Gevrey asymptotics for singularly perturbed q-difference-differential problems with an irregular singularity*, Abstract and Applied Analysis, vol. 2012, Article ID 860716.

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[17] A. LASTRA, S. MALEK, J. SANZ, *On Gevrey solutions of threefold singular nonlinear partial differential equations*, J. Differential Equations 255 (2013), no. 10, 3205–3232.

[18] A. LASTRA, S. MALEK, J. SANZ, *Summability in general Carleman ultraholomorphic classes*, J. Math. Anal. Appl. 430 (2015), no. 2, 1175–1206.

[19] A. LASTRA, J. SANZ, *Quasi-analyticity in Carleman ultraholomorphic classes*, Ann. Inst. Fourier (Grenoble) 60 (2010), no. 5, 1629–1648.

[20] S. MALEK, *Complex singularity analysis for linear partial q-difference-differential equations using nonlinear differential equations*, J. Dynam. Control. Syst. 19, no. 1 (2013), 69–93.

[21] S. MANDELBROJT, *Séries adhérentes, régularisation des suites, applications*, Collection de monographies sur la théorie des fonctions, Gauthier-Villars, Paris, 1952.

[22] S. MICHALIK, *Summability and fractional linear partial differential equations*, J. Dyn. Control Syst. 16 (2010), no. 4, 557–584.

[23] S. MICHALIK, *Analytic solutions of moment partial differential equations with constant coefficients*, Funkcialaj Ekvac. 56 (2013), 19–50.

[24] S. MICHALIK, *Analytic and summable solutions of inhomogeneous moment partial differential equations*, Funkcialaj Ekvac. 60, no. 3 (2017), 325–351.

[25] J.-P. RAMIS, *Dévissage Gevrey*, Astérisque 59–60 (1978), 173–204.

[26] J.-P. RAMIS, *Les séries k-sommables et leurs applications*, in: Lecture Notes in Phys. 126, Springer-Verlag, Berlin, 1980, 178–199.

[27] J.-P. RAMIS, Y. SIBUYA, *A new proof of multisummability of formal solutions of non linear meromorphic differential equations*, Ann. Inst. Fourier (Grenoble) 44 (1994), 811–848.

[28] J.-P. RAMIS, C. ZHANG, *Développement asymptotique q-Gevrey et fonction thêta de Jacobi*, C. R. Math. Acad. Sci. Paris 335 (2002), no. 11, 899–902.

[29] B. R. SALINAS, *Funciones con momentos nulos*, Rev. Acad. Ci. Madrid 49 (1955), 331–368.

[30] J. SANZ, *Flat functions in Carleman ultraholomorphic classes via proximate orders*, J. Math. Anal. Appl. 415 (2014), 623–643.

[31] C. ZHANG, *Transformations de q-Borel-Laplace au moyen de la fonction thêta de Jacobi*, C. R. Acad. Sci. Paris Sér. I Math. 331 (2000), 31–34.

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