ON RAMANUJAN'S PRIME COUNTING INEQUALITY

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Abstract. In this paper, we give a new upper bound for the number $N_{\mathscr{R}}$ which is defined to be the smallest positive integer such that a certain inequality due to Ramanujan involving the prime counting function $\pi(x)$ holds for every $x \geqslant N_{\mathscr{R}}$.

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