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EXTENSIONS OF DEMOCRACY–LIKE PROPERTIES FOR SEQUENCES WITH GAPS

MIGUEL BERASATEGUI AND PABLO M. BERNÁ*

Abstract. In [16], T. Oikhberg introduced and studied variants of the greedy and weak greedy algorithms for sequences with gaps. In this paper, we extend some of the notions that appear naturally in connection with these algorithms to the context of sequences with gaps. In particular, we will consider sequences of natural numbers for which the inequality $n_{k+1} \leqslant \mathbf{C}n_k$ or $n_{k+1} \leqslant \mathbf{C}+n_k$ holds for a positive constant $\mathbf{C}$ and all k, and find conditions under which the extended notions are equivalent their regular counterparts. In this context, we study an extension of democratic bases, proving that if $\mathbf{n} = (n_k)_{k=1}^{\infty}$ is an increasing sequence of natural numbers such that either $(n_{k+1}-n_k)_{k=1}^{\infty}$ is bounded and $(\mathbf{e}_k)_{k=1}^{\infty}$ is a Markushevich basis or $(n_{k+1}/n_k)_{k=1}^{\infty}$ is bounded and $(\mathbf{e}_k)_{k=1}^{\infty}$ is a Schauder basis, then $\mathbf{n}$-democracy is equivalent to democracy. Additionally, we give examples proving that these results are optimal, and we obtain similar results for some of the other properties that appear naturally in the study of the greedy algorithm.

Mathematics subject classification (2020): 41A65, 41A46, 46B15, 46B45.

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