Mathematical
Inequalities
& Applications
Volume 25, Number 4 (2022), 1191–1200
doi:10.7153/mia-2022-25-73

LOWER BOUND OF HAUSDORFF OPERATORS ON THE POWER WEIGHTED HARDY SPACES

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Abstract. Let $\alpha > -1$ and let φ be a measurable function on $(0,\infty)$ such that $\int_0^\infty t^\alpha |\varphi(t)|dt < \infty$. Denote by $H^1_{|\cdot|^\alpha}(\mathbb{R})$ the power weighted Hardy space associated with the power weight $|x|^\alpha$ and $\mathscr{H}_\varphi$ the Hausdorff operator associated with the kernel φ. Recently, it was showed in [11] that there is a constant $C > 0$ such that

$$\|\mathscr{H}_\varphi\|_{H^1_{|\cdot|^\alpha}(\mathbb{R})\to H^1_{|\cdot|^\alpha}(\mathbb{R})} \leqslant C\int_0^\infty t^\alpha |\varphi(t)|dt.$$

In this paper, we give a lower bound of $\|\mathscr{H}_\varphi\|_{H^1_{|\cdot|^\alpha}(\mathbb{R})\to H^1_{|\cdot|^\alpha}(\mathbb{R})}$ by proving that

$$\left|\int_0^\infty t^\alpha \varphi(t)dt\right| \leqslant \|\mathscr{H}_\varphi\|_{H^1_{|\cdot|^\alpha}(\mathbb{R})\to H^1_{|\cdot|^\alpha}(\mathbb{R})} \leqslant \int_0^\infty t^\alpha |\varphi(t)|dt.$$

Mathematics subject classification (2020): 47B38, 42B30.
Keywords and phrases: Hausdorff operator, Hardy space, power weight, holomorphic function.

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