

ON GENERALIZED CSISZÁR (f, g) -DIVERGENCE WITH AN APPLICATION FOR \mathbf{p} -MAJORIZATION

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Abstract. In this note, we develop some ideas from [8]. We introduce and investigate generalized Csiszár (f, g) -divergence generated by a convex function f and a concave function g . We derive a Csiszár–Körner type inequality for such (f, g) -divergences. We also study some special cases of the obtained inequality. In particular, we give a result for \mathbf{p} -majorization.

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