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RADII PROBLEMS FOR THE FUNCTION $az^2J_\nu''(z)+bzJ_\nu'(z)+cJ_\nu(z)$

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Abstract. In this paper, for three different normalizations of the function

$$N_\nu(z)=az^2J_\nu''(z)+bzJ_\nu'(z)+cJ_\nu(z),$$

where J_ν is Bessel functions of the first kind of order ν, the radius of parabolic starlikeness and uniform convexity are determined. We also give some simple results according to special cases of the parameters.

Mathematics subject classification (2020): 30C45, 33C15, 33C10.

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