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BORSUK'S PARTITION PROBLEM IN ℓ_p^4

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Abstract. In 1933, K. Borsuk made a conjecture that every n-dimensional bounded set can be divided into $n+1$ subsets of smaller diameter. Up to now, the problem is still open for $4 \leqslant n \leqslant 63$. In this paper, we study the generalized Borsuk's partition problem in ℓ_p^4 and prove that all bounded sets X in every ℓ_p^4 can be divided into 2^4 subsets of smaller diameter.

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