

WEIGHTED HARDY INEQUALITY WITH TWO-DIMENSIONAL RECTANGULAR OPERATOR: THE CASE $q < p$

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Abstract. A criterion is obtained for the boundedness of the two-dimensional rectangular integration operator from a weighted Lebesgue space $L_v^p(\mathbb{R}_+^2)$ to $L_w^q(\mathbb{R}_+^2)$ for $1 < q < p < \infty$, which is a supplement to E. Sawyer's theorem [8] and its extension [9] given for $1 < p \leq q < \infty$.

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