CONVEXITY PROPERTIES OF AREA INTEGRAL MEANS OVER THE ANNULI

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Abstract. For positive numbers t, p, q, c and an analytic function $f(z)$ in an annulus $R_1 < |z| < R_2$, let $M_{t,\varphi,q,c}(f,r)$ be the area integral means of f with respect to the weighted area measure $\varphi'(|z|^q)|z|^{q-2}\,dA(z)$, where $R_1 \leqslant c < R_2$. We show that $M_{t,\varphi,q,c}(f,r)^{\frac{1}{p}}$ is a convex function of r if f and φ satisfy certain conditions. The convexities of $\log M_{t,\varphi,q,c}(f,r)$ in r and $\log r$ can be obtained as special cases.

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