

COMMUTATORS GENERATED BY BMO-FUNCTIONS AND THE FRACTIONAL INTEGRALS ON ORLICZ-MORREY SPACES

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Abstract. We study the boundedness of commutators generated by BMO functions and the fractional integral operator on Orlicz-Morrey spaces of the second kind. To show this problem, we investigate the Fefferman-Stein type inequality concerning the Hardy-Littlewood maximal function and the sharp maximal operator and the boundedness of the Orlicz-fractional maximal operator in the Orlicz-Morrey spaces.

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