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THE PROOF OF A NOTABLE SYMMETRIC INEQUALITY

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Abstract. In this paper we give a proof of the inequality

$$\frac{1}{a_1^2+1}+\frac{1}{a_2^2+1}+\cdots+\frac{1}{a_n^2+1} \geqslant \frac{n}{2}$$

for nonnegative real numbers $a_1, a_2, \ldots, a_n$ satisfying

$$\sum_{1 \leqslant i<j \leqslant n} a_i a_j = \frac{n(n-1)}{2}.$$

The inequality is an equality for $a_1 = a_2 = \cdots = a_n = 1$, and also for $a_1 = a_2 = \cdots = a_{n-1} = \sqrt{\frac{n}{n-2}}$ and $a_n = 0$ (or any cyclic permutation).

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