Mathematical
Inequalities
& Applications
Volume 26, Number 4 (2023), 841–850 doi:10.7153/mia-2023-26-51

ASYMPTOTICS FOR GENERATING FUNCTIONS OF THE FUSS–CATALAN NUMBERS

ASSIS AZEVEDO* AND DAVIDE AZEVEDO

Abstract. We consider a certain class of polynomials with coefficients in $\mathbb{Z}_M$, all of which admit a unique zero. We prove that the zero of each of those can be given by a (multiple) sum involving the coefficients and a vectorial generalization of the Fuss-Catalan numbers.

We also consider the sequence of the partial sums of the generating function of the d-Fuss-Catalan numbers. Using the holonomy of this sequence, we study its asymptotic behaviour. The main difference from the known case $d = 2$ is, in that one, we have a "closed" expression for the generating function.

Mathematics subject classification (2020): 41A60, 05A10.
Keywords and phrases: Fuss-Catalan numbers, asymptotic behavior, holonomic sequence, polynomial congruence, vectorial Catalan numbers.

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