

OPERATORS ON HERZ–MORREY SPACES WITH VARIABLE EXPONENTS

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Abstract. This paper studies the nonlinear operators on Herz–Morrey spaces with variable exponents. We obtain our results by extending the extrapolation theory of Rubio de Francia to Herz–Morrey spaces with variable exponents. As applications of our main results, we obtain the boundedness of the spherical maximal functions, the nonlinear commutators of Rochberg and Weiss and the geometrical maximal operators on the Herz–Morrey spaces with variable exponents.

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